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An Unknotting Invariant for Welded Knots

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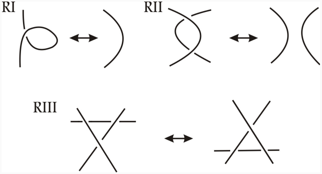
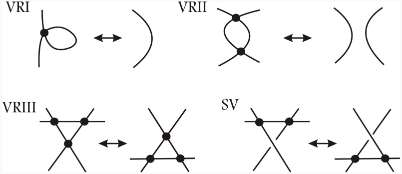
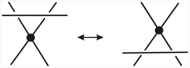
Based on the preprint joint with K. Kaur, A. Gill, and M. Prabhakar

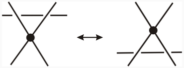
Welded Reidemeister moves





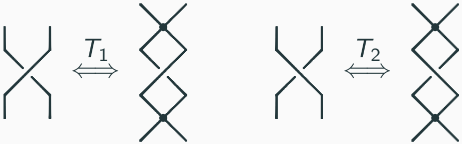
Classical Reidemeister moves and Virtual Reidemeister moves





Forbidden moves F_1 and F_2

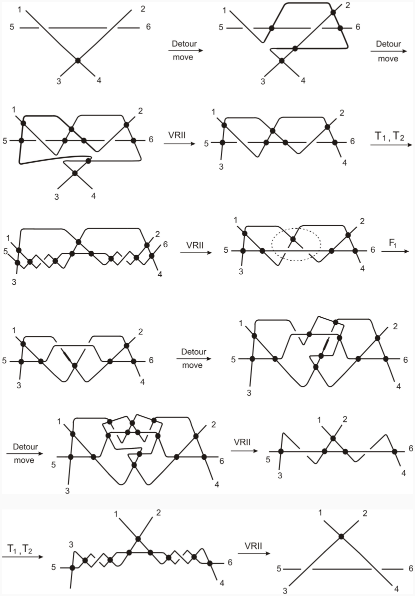
Twist move



Twist move of type T_1 and type T_2.

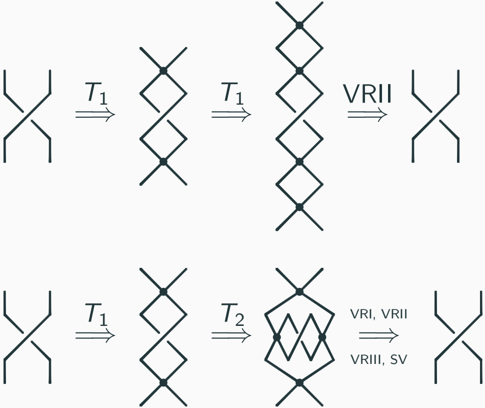
Theorem. Twist move is an unknotting operation for welded knots.

Realization of F_2-move



Composition of two twist moves

Proposition. If D' is a diagram obtained from a welded knot diagram D by applying a twist move twice at the same crossing $c \in C(D)$, then D' is equivalent to D.



Unknotting twist number

The unknotting twist number $ut(K)$ is the minimum number of twist moves required, taken over all welded knot diagrams representing K, to convert K into the trivial welded knot,

$$ut(K) = \min\{ut(D) \mid D \in [K]\}.$$

Let $\mathbf{b}(2n + \frac{1}{2})$ be a two-bridge knot with the rational parameter $2n + \frac{1}{2}$, for integer $n \geq 1$.

Proposition For any integer $n \geq 1$ we have $ut(\mathbf{b}(2n + \frac{1}{2})) = 1$.

Welded unknotting number

The welded unknotting number $u_w(K)$ is the minimum number of classical crossings to welded crossings changes, required, taken over all welded knot diagrams representing K, to convert K into the trivial welded knot,

$$u_w(K) = \min\{u_w(D) \mid D \in [K]\}.$$

Warping degree

The warping degree was introduced by A. Shimizu (2009, 2010) for classical knots and links.

Let D be an oriented welded knot diagram of K, choose a non-crossing point a on D (based point). The warping degree $d(D_a)$ of D_a is the number of classical crossings encounter first at under crossing point while starting from a and traverse along the orientation of D_a.

The warping degree $d(D) = \min\{d(D_a \,|\, a \in D\}$. Let $-D$ be the inverse of D. The warping degree of a knot K is

$$d(K) = \min\{d(D), d(-D) \mid D \in [K]\}.$$

Low and upper bounds

Theorem. If K is a welded knot, then

$$\frac{1}{2}u_w(K) \leq ut(K) \leq d(K).$$

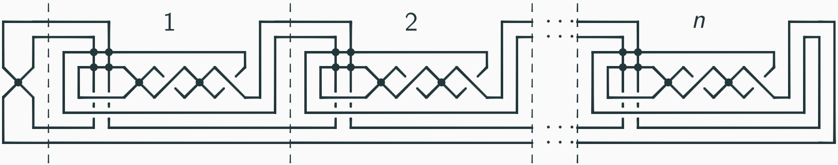
Twist distance

Define a twist-distance $d_T(K, K')$ between two welded knots K and K' as the minimum number of twist moves required to convert D into D', where minimum is taken over all diagrams D of K and D' of K'.

Gordian complex $\mathcal{G}_T$ of welded knots by twist move is defined by considering the set of all welded knot isotopy classes as vertex set of $\mathcal{G}_T$ and a set of welded knots $\{K_0, ..., K_n\}$ spans an n-simplex if and only if $d_T(K_i, K_j) = 1$ for all $i \neq j \in \{0, 1, \ldots, n\}$.

Gordian complex

Theorem. The Gordian complex $\mathcal{G}_T$ contains an infinite family of welded knots $\{WK_n\}_{n\geq 0}$ satisfying $d_T(WK_m, WK_n) \leq 1$ for distinct integer $m, n \geq 0$.



Welded knot WK_n.

Applying of twist move.

Question. Are all welded knots WK_n distinct?

Thank you!